3D shape from the structure of pencils of planes and geometric constraints

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Abstract. Active stereo systems using structured light has been used as practical solutions for 3D measurements. Among them, calibrated line lasers (*i.e.* , calibrated planes of light) are commonly used as structured light in approaches known as light section methods. On the other hand, several 3D reconstruction methods based on uncalibrated planes of light have been also proposed. In these methods, 3D curves lit by the planes of light (*i.e.* , intersections of the scene and the planes of light) are detected in 2D images as 2D curves, and the 3D shapes of the 3D curves and the 3D planes are reconstructed using the detected intersections of these 2D curves. In this paper, we propose a novel 3D reconstruction method under an assumption that the planes of light forms several pencils of planes (*i.e.* , groups of planes each of which shares a single line). This paper shows two typical cases where the proposed method can be applied; one is shape from cast-shadow problems where two known straight edges generate the shadows, and the other is 3D reconstruction for one-shot range scanner based on projector-camera systems where grid patterns are projected from the video projectors.

1 Introduction

Active stereo systems using structured light has been used as practical solutions for 3D measurements. One of the commonly-used geometries of structured light is planes. Such a type of structure can be generated as lights projected by line lasers or shadow boundaries cast from straight edges, since both features of light spread within 3D planes (we refer to such planes as light planes). If the light planes are calibrated, the structured light can be used for triangulation. In the approaches known as light section methods, positions of the light planes are either known in advance or estimated by other means, then, 3D information of the curves detected as intersections between the light planes and the scene (we refer to such curves as light curves) are reconstructed by triangulation.

Recently, a different approaches for 3D reconstruction using light planes whose positions are not known in advance (*i.e.* , the light planes are uncalibrated) [1–3] are proposed. It was already pointed out that, if uncalibrated light planes are used, the 3D information of the scene can be solved, at most, with remaining indeterminacy of a 4-DOF family of generalized projective bas-relief (GPBR) transformations [4]. As a variation of these approaches, 3D scene reconstruction using multiple light planes projected from a video projector using a projector-camera system was also proposed [5]. However, since this method solves the position of light planes by representing each of the planes with 3 parameters, computational cost rapidly increases as the number of light planes increases. This also causes instability of numerical calculations since the number of variables increases. Thus, dense reconstruction was not achieved.

One of the efficient method for stabilizing estimation of the light planes is that reducing the total number of parameters of the planes by giving some constraints to them. For example Bouguet *et al.* assumed a fixed light source and used a constraint that all the light planes share the same 3D position, because all the planes are generated as shadow boundaries cast from the single light source [1, 6]. Using the constraint, each of the planes can be represented by 2 parameters, which results in reduced computational complexity and improved numerical stability.

In this paper, reduction of the number of the parameters of planes is further utilized. This is achieved by using a geometrical structure called a pencil of planes, which is a group of planes that share a fixed line (Fig. 1(left)). Such a geometrical structure appears on, for example, shape from cast-shadow problems where shadows are generated by fixed buildings (Fig. 1(right)), or one-shot 3D scanning using a projector-camera system where the projector projects a grid pattern to the scene (Fig. 1(middle)). Generally, a plane can be represented by 3 parameters, however a plane included by a known pencil of planes can be represented by one parameter. In this paper, by using the 1-parameter representation, the number of variables required for scene reconstructions is reduced significantly, and, thus, the numerical stability of the solution is improved. Furthermore, by using the representation, relationships between planes of the optimal solution can be described by simple equations, which results in equations with even less variables that can be solved more stably.

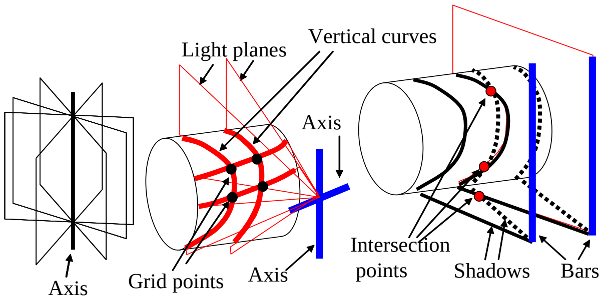


Fig. 1. Left:Pencil of planes. Middle:Example of shape from cast-shadows. Right:Example of one-shot active stereo.

In this paper, to show the effectiveness of the method, we tested the method by two typical applications, such as shape from cast-shadow using shadows of two straight edges, and a one-shot active stereo system based on a projector-camera system that uses structured light of a grid pattern.

The former application is a method for 3D reconstruction from cast-shadows where two known straight edges cast shadows to the scene with unknown light positions. This can be applied for the case that, for example, there is a building having two straight edges and the shadows of the edges are used for 3D reconstruction.

The latter application is a one-shot range finder using a projector-camera system. When a grid pattern is emitted from a projector, the light planes formed by the vertical and the horizontal lines of the grid shares a vertical and a horizontal lines that passes the optical center of the projector, respectively. Thus, these light planes are elements of a pencil of planes. By using these properties, 3D reconstruction using the grid points can be solved using the proposed method.

2 Related work

There are many existing commercial active stereo products that use calibrated light planes. [7, 8]. For these systems, light planes are pre-calibrated and controlled precisely. On the other hand, some researchers proposed that projecting light planes (either by line lasers or by cast-shadows) to the scene and reconstruct 3D information from detected positions of the intersection points between the light curves. Bouguet *et al.* proposed a method which allowed users to move a straight edged object freely so that the shadow generated by a fixed light source sweep the object [1, 6]. The technique requires calibration of camera parameters, a light source position, and a reference plane. Their method uses 2-parameter representation of planes using assumption of a known point light source. If k intersection points are detected for m light planes, solutions up to 3-DOF indeterminacy can be obtained by solving k simultaneous equations that have $2m$ variables. The remaining 3 DOFs are determined using 3 points on a known reference plane.

Kawasaki *et al.* and Kutulakos *et al.* projected line lasers to the scene and reconstructed the 3D information of the light curves [9, 3]. In these methods, the light planes are formulated by 3-parameter representations. If k intersection points are detected for m light planes, solutions up to 4-DOF indeterminacies can be obtained by solving k simultaneous equations that have $3m$ variables. The 4-DOF indeterminacies are determined by using geometrical relationships between the planes such as angles [2].

To generate light planes, video projector can be also used. Koninckx *et al.* proposed a technique allowing dense shape reconstruction based on a single image using a set of stripes [10]. This was achieved by combining dense unidentified stripes and several identified stripes. Their method depends on relative numbering of the dense patterns, which assumes local smoothness of the surface and may be disturbed by shape discontinuities and line detection failures. Similarly, Frueh and Zakhor [11] used vertical stripes and a horizontal line. The vertical stripes are used for shape reconstruction and the horizontal line for identifying the vertical stripes. Kawasaki *et al.* proposed a method of shape reconstruction from a single grid pattern [5]. Their method is based on coplanar constraint[9, 2, 3] instead of using spatial coding by complex patterns or colors. The advantage is that it does not assume smoothness or uniform reflectance of surfaces and is robust against local discontinuities of grid patterns. The disadvantage is however that the object surface should be sufficiently large to use coplanar constraint effectively, which is one of the issue to be improved in this paper.

3 3D shape from multiple pencils of planes

3.1 Problem definition

Structured light generated by line lasers or straight pattern projected by a video projector forms light planes in 3D space. Light planes are also generated as boundaries between shadow and non-shadow regions where shadows are generated by straight edges.

A pencil of planes is a group of planes that share a fixed 3D line. The structure of a pencil of planes often appears in the problems of 3D reconstruction using structured light, or in the shape from cast-shadow problems. Some of the examples are already described, such as light planes generated by parallel line patterns projected from a video projector, or light planes of cast shadows with a fixed straight edge and a moving light source.

In this paper, we assume that there are two pencils of planes and they share a single plane. The axes (the lines that are shared by the planes) are assumed to be known. An example case is that there are two parallel bars whose position is known, and the cast shadows of the bars generated by unknown moving light source are used for 3D reconstruction. The cast shadow form a light plane, and the light plane moves as the light source moves. Since the light planes share a line that corresponds to the bar, they are elements of a pencil of planes. If the two bars are parallel, the two pencils of planes share a single plane, which fulfills the assumption.

Another example case is that a video projector projects a grid pattern. Here, light planes generated by the parallel vertical lines are elements of a pencil of planes. The group of horizontal lines also form a pencil of planes. The axes of the two pencils of planes are vertical and horizontal line that goes through the optical center of the projector and are parallel with the vertical and horizontal lines on the image plane respectively. Since these axes are on a single plane, these pencils of planes share the plane, which fulfills the assumption.

The light planes generate detectable patterns on the scene, which are the intersection curves between the light planes and the surface of the scene and are called light curves in this paper. The light curves are detected by the camera. Here, intersection points between multiple light curves can be used as constraints to reconstruct 3D information of the light planes. Since each of the light planes is an element of a pencil of planes, it can be represented by a single parameter as described in the following section.

3.2 Representation of planes

In the following sections, the proposed 3D reconstruction method is explained using an example application of one-shot range finder based on a projector-camera system where a grid pattern is emitted from the projector.

First, representation of the light planes are explained. In the case of the example, the set of vertical light planes is a pencil of planes that share one straight line that goes though the optical center of the projector. The set of horizontal planes is also a pencil of planes.

Each plane is represented by 3 parameters in [5]; however, since the vertical planes are pencil of planes, each of them can be represented by 1 parameter [1]. This reduces the number of variables to represent the vertical and horizontal planes, and makes the numerical calculations much more efficient. The formulations are described in the following. We assume both vertical and horizontal planes does not include the optical center of the camera. Thus, a vertical plane can be represented as

$$v_1 x_1 + v_2 x_2 + v_3 x_3 + 1 = \mathbf{v} \cdot \mathbf{x} + 1 = 0, \tag{1}$$

where $\mathbf{v} \equiv (v_1, v_2, v_3)$ is a parameter vector of the plane and $\mathbf{x} \equiv (x_1, x_2, x_3)$ is a 3D point represented by the camera coordinates. All the vertical planes include the axis of the pencil of planes. Let a point on the axis be $\mathbf{p}_v \equiv (p_{v1}, p_{v2}, p_{v3})$. Let the direction vector of the line (the vertical direction of the projector) be $\mathbf{q} \equiv (q_1, q_2, p_3)$. Then, $\mathbf{v}$ should fulfill

$$v_1 p_{v1} + v_2 p_{v2} + v_3 p_{v3} + 1 = \mathbf{v} \cdot \mathbf{p} + 1 = 0 \tag{2}$$

$$v_1 q_1 + v_2 q_2 + v_3 q_3 = \mathbf{v} \cdot \mathbf{q} = 0. \tag{3}$$

By solving these equation with respect to $\mathbf{v}$, a general solution of 1DOF

$$\mathbf{v} = \mathbf{v_0} + \eta(\mathbf{p}_v \times \mathbf{q}) \tag{4}$$

is obtained, where $\mathbf{v_0}$ is a parameter vector of an arbitrary vertical plane (*i.e.* , an arbitrary plane that fulfill Eqs. (2) and (3)). Similarly, a parameter vector of a horizontal plane $\mathbf{h} \equiv (h_1, h_2, h_3)$ can be represented by

$$\mathbf{h} = \mathbf{h_0} + \rho(\mathbf{p}_h \times \mathbf{r}) \tag{5}$$

where $\mathbf{r}$ is the horizontal direction of the projector, $\mathbf{p}_h$ is one of the points on the axis of the pencil of planes that include the horizontal planes, and $\mathbf{h_0}$ is a parameter vector of an arbitrary horizontal plane. The plane that goes through the optical center of the projector and is parallel with the projector's image plane is called the focal plane of the projector [12]. The focal plane of the projector is shared by the two pencils of planes. Thus, by letting the parameter of the plane that is shared by the two pencils of planes (in this case, the focal plane of the projector) be $\mathbf{s}$, and by defining $\mathbf{v}' \equiv \mathbf{p}_v \times \mathbf{q}$, and $\mathbf{h}' \equiv \mathbf{p}_h \times \mathbf{r}$, vertical planes and horizontal planes can be represented by the following equations:

$$\mathbf{v} = \mathbf{s} + \eta\mathbf{v}', \quad \mathbf{h} = \mathbf{s} + \rho\mathbf{h}'. \tag{6}$$

[1] A plane in the 3D space corresponds to a point in the dual space. A pencil of planes in the 3D space corresponds to a line in the dual space. 1 parameter representation of the planes corresponds to representing points on a line using 1 parameter in the dual space.

3.3 Solution

If a grid point (*i.e.* , intersection points made by a vertical and and horizontal curve) is detected, and the vertical and horizontal curves are respectively included by a vertical plane $\mathbf{v}$ and horizontal plane $\mathbf{h}$, then the following equation is fulfilled:

$$\mathbf{u} \cdot (\mathbf{v} - \mathbf{h}) = 0 \tag{7}$$

where $\mathbf{u} \equiv (u, v, 1)$ represents the 2D position of the detected grid point (u, v) in the normalized camera coordinates. From Eqs. (6) and (7),

$$\eta(\mathbf{u} \cdot \mathbf{v}') = \rho(\mathbf{u} \cdot \mathbf{h}') \tag{8}$$

is obtained. This is a simple constraint that the ratio of the parameter η of the vertical plane and the parameter ρ of the horizontal plane is determined from the grid point (u, v).

Each of the detected vertical curves is included by a vertical plane. Let the index of a detected vertical curve be i and the parameter of the corresponding vertical plane be η_i. Similarly, let the index of a horizontal curve and the plane parameter of the corresponding plane be j and ρ_j, respectively. If those curves has a grid point $\mathbf{u}_{i,j}$, then $\eta_i(\mathbf{u}_{i,j} \cdot \mathbf{v}') = \rho_j(\mathbf{u}_{i,j} \cdot \mathbf{h}')$. By defining constants $F_{i,j} \equiv \mathbf{u}_{i,j} \cdot \mathbf{v}'$ and $G_{i,j} \equiv \mathbf{u}_{i,j} \cdot \mathbf{h}'$,

$$F_{i,j}\eta_i = G_{i,j}\rho_j \tag{9}$$

is obtained. By accumulating Eq. (9) for all the grid points, L simultaneous equations with $(M + N)$ variables $(\eta_i, \rho_j, 1 \leq i \leq M, 1 \leq j \leq N)$ are obtained, where M and N are the numbers of detected vertical and horizontal curves, respectively.

Let k be an index of a grid point, and let $i(k)$ and $j(k)$ be the indices of the vertical and horizontal planes that go through grid point k. Let $\mathbf{T}$ be a $L \times M$ matrix whose (p, q) element is $F_{i(p),j(p)}$ if $q = i(p)$, and otherwise 0. Let $\mathbf{R}$ be a $L \times N$ matrix whose (p, q) element is $G_{i(p),j(p)}$ if $q = j(p)$, and otherwise 0. Then, by defining $\boldsymbol{\eta} \equiv (\eta_1, \cdots, \eta_M)^\top$ and $\boldsymbol{\rho} \equiv (\rho_1, \cdots, \eta_N)^\top$, simultaneous Eq. (9) are represented by

$$\mathbf{T}\boldsymbol{\eta} = \mathbf{R}\boldsymbol{\rho}. \tag{10}$$

To solve Eq. (10) by least squares method,

$$||\mathbf{T}\boldsymbol{\eta} - \mathbf{R}\boldsymbol{\rho}||^2 = \left\| [\mathbf{T}| - \mathbf{R}] \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\rho} \end{bmatrix} \right\| \tag{11}$$

should be minimized. This can be achieved by calculating the eigenvector associated to the minimum eigenvalue of symmetric $(M + N) \times (M + N)$ matrix $[\mathbf{T}| - \mathbf{R}]^\top[\mathbf{T}| - \mathbf{R}]$. There are efficient numerical algorithms for this problem.

It is possible to reduce variables from Eq. (11). Solution of least squares method is obtained by

$$\min_{\boldsymbol{\eta},\boldsymbol{\rho}} ||\mathbf{T}\boldsymbol{\eta} - \mathbf{R}\boldsymbol{\rho}||^2 = \min_{\boldsymbol{\rho}}(\min_{\boldsymbol{\eta}} ||\mathbf{T}\boldsymbol{\eta} - \mathbf{R}\boldsymbol{\rho}||^2). \tag{12}$$

$\min_{\boldsymbol{\eta}} ||\mathbf{T}\boldsymbol{\eta} - \mathbf{R}\boldsymbol{\rho}||^2$ is achieved when $\boldsymbol{\eta} = \mathbf{T}^{\dagger}\mathbf{R}\boldsymbol{\rho}$, where $\mathbf{T}^{\dagger} \equiv (\mathbf{T}^{\top}\mathbf{T})^{-1}\mathbf{T}^{\top}$ is a pseudo inverse of $\mathbf{T}$. Since (r, c) element of $\mathbf{T}^{\top}\mathbf{T}$ is

$$(\mathbf{T}^{\top}\mathbf{T})_{r,c} = \sum_s \mathbf{T}^{\top}_{r,s}\mathbf{T}_{s,c} = \sum_s \mathbf{T}_{s,r}\mathbf{T}_{s,c} \tag{13}$$

$$= \begin{cases} \sum_s T^2_{s,r} & \text{if} \quad r = c \\ 0 & \text{otherwise} \end{cases}, \tag{14}$$

$\mathbf{T}^{\top}\mathbf{T}$ is a diagonal matrix (equal of Eq. (14) comes from the fact that each row of $\mathbf{T}_{s,c}$ has only one non-zero element). Thus, $(\mathbf{T}^{\top}\mathbf{T})^{-1}$ can be directory calculated and $\mathbf{T}^{\dagger}$ is obtained by simple multiplication. By substituting $\boldsymbol{\eta} = \mathbf{T}^{\dagger}\mathbf{R}\boldsymbol{\rho}$ into Eq. (12),

$$\min_{\eta,\rho} ||\mathbf{T}\boldsymbol{\eta} - \mathbf{R}\boldsymbol{\rho}||^2 = \min_{\rho}(||\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R}\boldsymbol{\rho} - \mathbf{R}\boldsymbol{\rho}||^2) \tag{15}$$

$$= \min_{\rho}(||(\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R} - \mathbf{R})\boldsymbol{\rho}||^2) \tag{16}$$

is obtained. This means that the optimal value of $\boldsymbol{\rho}$, which we term by $\hat{\rho}$, is calculated as the eigenvector associated to the minimum eigenvalue of matrix $(\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R} - \mathbf{R})^{\top}(\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R} - \mathbf{R})$. Then the optimal value $\hat{\eta}$ is obtained by $\hat{\eta} = \mathbf{T}^{\dagger}\mathbf{R}\hat{\rho}$. Since $(\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R} - \mathbf{R})^{\top}(\mathbf{T}\mathbf{T}^{\dagger}\mathbf{R} - \mathbf{R})$ is a $N \times N$ symmetric matrix, the computational complexity can be further reduced.

Eigenvectors always has an ambiguity of scaling. Actually, if Eq. (10) is fulfilled, $\boldsymbol{\eta}$ and $\boldsymbol{\rho}$ can be replaced by $c\boldsymbol{\eta}$ and $c\boldsymbol{\rho}$, respectively. Inversely, if all the vertical and horizontal curves are connected by one or more grid points, then, this equation does not have indeterminacy except for scaling. So, 3D reconstruction is performed for each of the connected groups of the curves.

This one DOF indeterminacy of scaling cannot be determined, if only position of intersection points are used as constraints. Thus, additional geometrical constraints depending on the specific applications should be used to achieve Euclidean reconstruction, for example, orthogonality between two pencils; it will be explained in the next section in detail.

Advantages of the proposed method can be summarized as the follows.

1. 3D information of the light curves is restored except for 1-DOF indeterminacy from only the intersection points between the curves. The remaining 1-DOF indeterminacy can be determined from additional constraints such as geometrical constraints of the scene or those between light planes.
2. Since the light planes are assumed to be elements of known pencils of planes, each of them can be represented by one parameter. By using this representation, scene reconstruction can be solved by using linear equations with less variables compared to the previous approaches. Thus,
   (a) the numerical solution is more efficient and stable,
   (b) and more dense reconstruction is practically achieved.

4 Examples of applications

Here, two typical examples of applications of the proposed method will be shown. One is shape from cast-shadows using two fixed straight edges, and the other is a one-shot, structured-light range scanner using a video projector that projects a grid pattern.

4.1 Shape from cast-shadows

Straight edges that exist on a single plane can be found everywhere in the real world. The proposed method can be applied to solve a shape from cast-shadow problem which uses such a pair of edges as occluder for shadows from an uncalibrated moving light source. In this case, the parameters of the plane that includes both axes of the two pencils of planes can be used as vector $\mathbf{s}$ of Eq. (6).

A solution by the proposed method has 1-DOF indeterminacy. Several methods can be considered for determining this ambiguity. For example, if the position of one of the intersections between the light curves are known, the ambiguity can be solved using this constraint. Another example is orthogonality between two planes in the scene. In actuality, most of other constraints that are related to scales or angles can also be used for the purpose.

4.2 One-shot scanner using a structured light of a grid pattern

We have used the proposed method to develop a one-shot range scanner using a structured light of a grid pattern composed of a camera and a projector. In this application, the grid lines emitted by the projector are assumed to be known. However, the actual IDs of the detected curves captured by the camera are not known. Thus, triangulation cannot be processed directly from the the detected curves. Even in this case, the proposed method can reconstruct 3D information from the grid points (intersection points between the vertical and horizontal lines) up to 1-DOF ambiguity. Then, the remaining 1-DOF ambiguity can be solved by matching process between group of the light planes of the solution with the 1-DOF ambiguity and the known positions of the planes that are calculated from the locations of the grid lines. This matching algorithm is as follows.

The solutions of the light planes obtained by the proposed method can be described as $\boldsymbol{\eta} = c\hat{\eta}$ and $\boldsymbol{\rho} = c\hat{\rho}$, where c is an arbitrary scalar constant. Here, the value of c cannot be determined from Eq. (10), however, if the true value of c is $\bar{c}$, then the vertical and horizontal light planes of solutions $\bar{c}\boldsymbol{\eta}$ and $\bar{c}\boldsymbol{\rho}$ should be coincide with the given grid pattern which is actually projected. Thus, matching between the 1-DOF solution and the light planes that are actually projected can be used for solve the 1-DOF indeterminacy.

For simplicity, we calculate the similarity between the solution of "horizontal" light planes calculated from grid points and the horizontal lines of the given grid pattern that is actually emitted. Let P be the number of the horizontal line of the given grid and let $\mathbf{g}_l, 1 \leq l \leq P$ be the parameter vectors of the horizontal planes that are associated to the lines. These parameter vectors can be calculated in advance. Then, the matching error function to compare $c\rho_j, 1 \leq j \leq N$ and $\mathbf{g}_l, 1 \leq l \leq P$ can be defined as $E(c) = \sum_{1 \leq i \leq N} \min_{1 \leq l \leq P} e(\mathbf{g}_l, \mathbf{s} + c\rho_j \mathbf{h}')$ where $e(\mathbf{a}, \mathbf{b})$ is a function that outputs the square of the angle between $\mathbf{a}$ and $\mathbf{b}$. By minimizing $E(c)$ with respect to c, $\bar{c}$ is obtained. Since $E(c)$ has numerous local minima, it is difficult to apply numerical optimization methods. In actual cases, the range of c is limited by the given horizontal planes, thus, full 1D search of the range by intervals of required precision is sufficient.

To increase robustness of the search of solution, using color information for matching is effective. For example, $e(\mathbf{a}, \mathbf{b})$ in the similarity function is calculated only if the light planes $\mathbf{a}$ and $\mathbf{b}$ have the same color, and otherwise outputs a large number. This reduces false matching and the matching process is stabilized. To maintain the advantage of just using a small number of colors for stable image processing, using de Bruijn sequence is another option. In our experiment, we implemented the latter method.

5 Experiments

5.1 Shape from cast-shadows (simulated data)

To validate the proposed method, shape from cast-shadows using two parallel straight edges were examined using a simulated data. The data synthesized as a simulation is shown in Fig. 2(left). In the data, the positions of the two straight edges that generate shadows were given in advance. The shadow boundaries were also given as the light curves. To solve the ambiguity of the solution, we should provide additional constraints, *i.e.* , we used the fact that the "floor" of the scene was a plane that was orthogonal to the straight edges. To use this assumption, 3 intersection points that were on the floor was given. From the information, intersection points between the light curves are extracted and used as the inputs for the proposed method. By using the method, the solution with 1-DOF ambiguity is obtained. Then, the orthogonality between the straight edges and the plane that includes the given 3 points that should be on the floor are optimized by 1-D search. Fig. 2(middle) and (right) show the result of the reconstruction. From the figure, it can be confirmed that the reconstruction was accurately achieved.

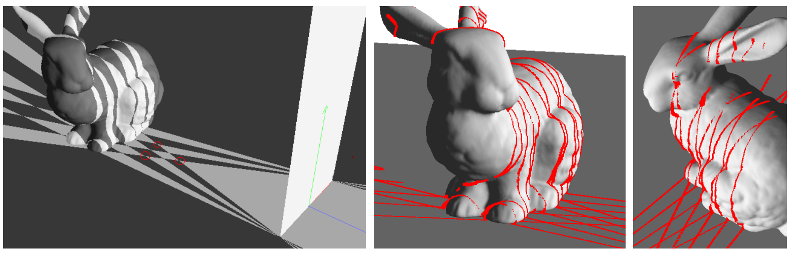
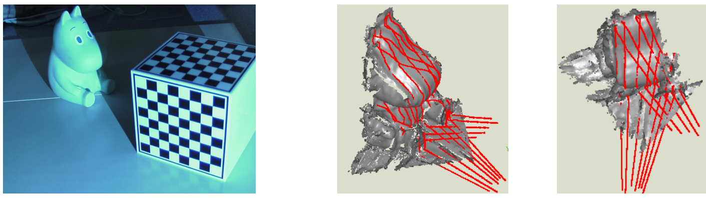


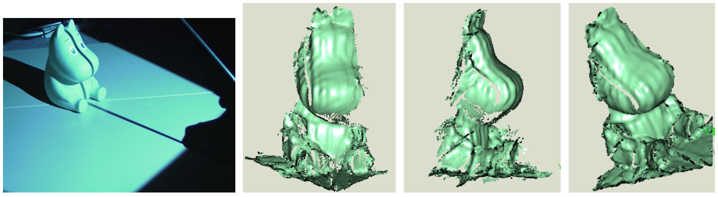
Fig. 2. Result of simulated data: (Left) The input data. Red circle are intersection points given to be on the floor. (Middle,right) The reconstruction result. The shaded model is the ground truth and the points are reconstructed light curves.

5.2 Shape from cast-shadows (real data)

Next, the shape from cast-shadow method was applied to real objects. Since the two straight edges should be known, a calibration box was captured in advance and two edges of the calibration box was used to generate shadows. The light source was moved while the shadows were captured. In this process, three of the intersection points that were on the floor (the edges to generate shadows were orthogonal to the floor) were given to solve the 1-DOF ambiguity. Fig. 3(a) shows the captured scene with cast shadow and a calibration box, and Fig. 3(b) and (c) show the reconstructed light curves (red curves in the figure).



(a) Capturing scene for sparse reconstruction. (b) Sparse reconstruction. (c) Sparse reconstruction from another viewpoint.

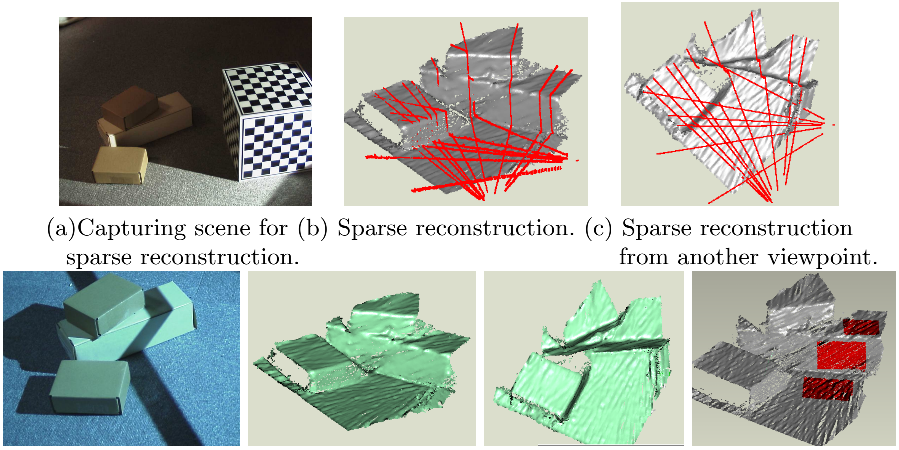


(d) Capturing scene for dense reconstruction. (e) Dense reconstruction from various view points.

Fig. 3. Shape from shadow (Results of a plastic toy).

Then, we used the reconstructed light curves to obtain dense shape data of the scenes. Fig. 3(d) shows the captured scene with cast shadow. After the above steps are processed, the scenes were scanned by shadows that were generated by a straight bar that was freely moved. While the scanning, the light curves were detected. Then, intersection points between the 3D reconstruction result and the new light curves were extracted. From these intersection points, the light plane for each of the new light curves were estimated using plane approximation using principal component analysis. By applying a triangulation method using the estimated light planes, dense reconstruction of the scenes were achieved as shown in Fig. 3(e).

We also evaluated our method by scanning rectangular boxes. The scanning scene and the results are shown in Fig. 4. To evaluate the accuracy, 3 planer regions of the reconstructed shape as shown in Fig. 4 (g) were extracted and each of them were fitted to a plane using principal component analysis. The RMS errors for the normal directions were $6.8 \times 10^{-4}(m)$, $9.5 \times 10^{-4}(m)$ and $7.4 \times 10^{-4}(m)$. There were two orthogonal angles between the 3 planes. To evaluate the angular errors, the two angles were calculated. The errors of the angles



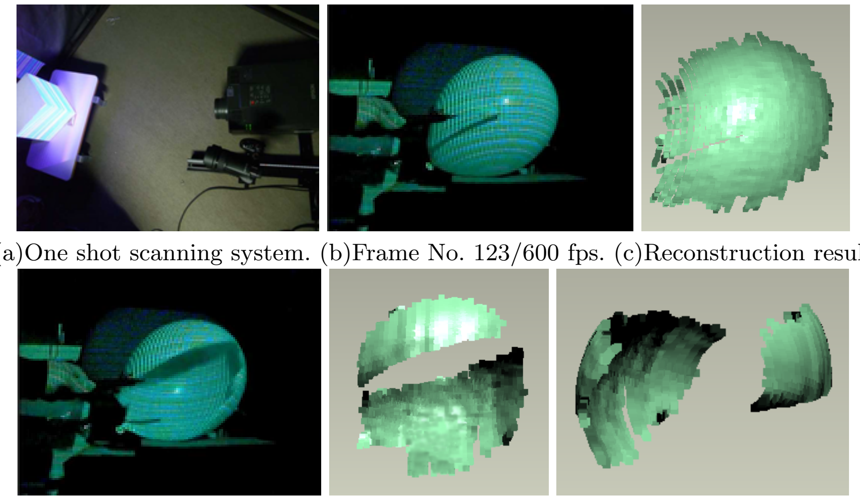
(a)Capturing scene for sparse reconstruction. (b) Sparse reconstruction. (c) Sparse reconstruction from another viewpoint.

(d)Capturing scene for dense reconst. (e) Dense reconst. (f) Dense reconst. from another view. (g) Selected faces for evaluation.

Fig. 4. Shape from shadow (Results of a box scene).

were 0.17° and 1.79°. Therefore, we can confirm that our method accurately reconstructed the shape.

5.3 One-shot active stereo



(a)One shot scanning system. (b)Frame No. 123/600 fps. (c)Reconstruction result

(d)Frame No. 124/600 fps. (e)Reconstruction result (f)Reconstruction result from another viewpoint

Fig. 5. Reconstruction of a exploding balloon by one shot scanner.

Next, we conducted a 3D shape reconstruction using a projector camera system(Fig. 5 (a)). Since the system requires just a single input image with a simple grid pattern, it has a good potential to capture a extremely fast moving object; note that this attribute is one of the important advantage of our method. To prove the ability of high frame-rate scanning, we captured an exploding balloon with 600fps with shutter speed 1/15000 (Fig. 5 (b) and (d)). The reconstruction results are shown in Fig. 5 (c), (e) and (f). We can confirm that the system successfully reconstructed the shape with the proposed method.

6 Conclusion

This paper proposed reconstruction method from the structure of pencils of planes. The planes that form two pencils of planes that share a single plane are assumed, and the 3D information of the planes except 1-DOF indeterminacy were reconstructed by the proposed method. Remaining 1-DOF indeterminacies can be stably solved by either constraint in the scene or known geometric relationship between pencil structures. Two typical applications of the method was shown; shape form cast-shadow and one-shot range finder based on a projector-camera system; and effectiveness of the method were confirmed with successful reconstruction results.

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